



NPTEL ONLINE CERTIFICATION COURSES

COMMUNICATIONS NETWORK

Dr. Goutam Das

G. S. Sanyal School of Telecommunications, IIT KHARAGPUR

Module 05 : Queuing Theory(contd.)

Lecture No 20 : Little's Theorem

Concepts Covered

Special Properties of Poisson Distribution

Residual Time Distribution

Little's Formula - Introduction

References

Book:

Digital Telephony – J. Bellamy

Queuing Systems Vol. 1 – L. Kleinrock

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Module 05 : Queuing Theory(contd.)

Lecture No 21 : Little's Theorem cont'd

Concepts Covered

Little's Formula – Derivation

Markov Chain-Introduction

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References

Book:

Digital Telephony – J. Bellamy

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Module 05 : Queuing Theory

Lecture No 22 : DTMC

Concepts Covered

Discrete Time Markov Chain

Homogeneous Markov Process

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References

Book:

Digital Telephony – J. Bellamy

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Module 05 : Queuing Theory

Lecture No 23 : DTMC

Concepts Covered

Multi-stage Markov Transition

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References

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Module 05 : Queuing Theory

Lecture No 24 : DTMC To CTMC

Concepts Covered

Chapman Kolmogorov Forward Equation

DTMC to CTMC - Probability to Rate Matrix

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References

Book:

Digital Telephony – J. Bellamy

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Module 05 : Queuing Theory

Lecture No 25 : CTMC

Concepts Covered

Chapman Kolmogorov Forward Equation

DTMC to CTMC - Probability to Rate Matrix

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References

Book:

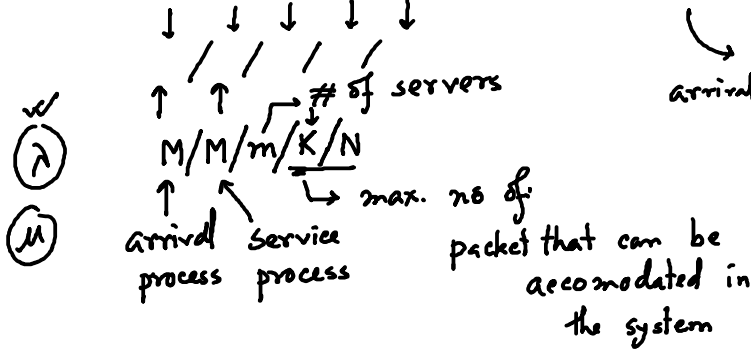
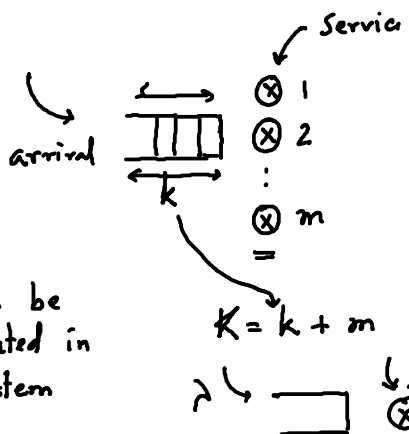
Digital Telephony – J. Bellamy

Queuing Systems Vol. 1 – L. Kleinrock

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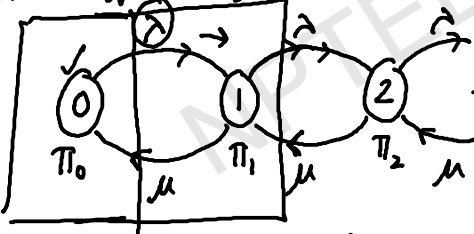
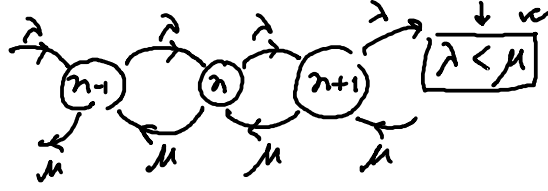
M/M/1

⇒ Kindall's notation



M/M/1/∞/∞

↳ Infinite buffer single server case



$$\begin{bmatrix} \lambda & 0 \\ 0 & \mu \end{bmatrix}$$

$$\frac{\lambda}{\mu} = \rho$$

$$\rho < 1$$

$$\lambda \pi_0 = \mu \pi_1 \Rightarrow \pi_1 = \left(\frac{\lambda}{\mu}\right) \pi_0 \Rightarrow \pi_1 = \rho \pi_0 \quad \boxed{\pi Q = 0}$$

$$\lambda \pi_1 = \mu \pi_2 \Rightarrow \pi_2 = \rho \pi_1 = \rho^2 \pi_0$$

$$\lambda \pi_{n-1} = \mu \pi_n \Rightarrow \pi_{n-1} = \rho^{n-1} \pi_0$$

$$\pi_n = \rho^n \pi_0$$

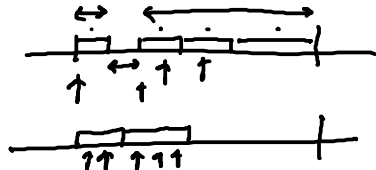
λ
of arrival
per unit time

μ → service rate

1/μ ⇒ average service time.

$$\sum_{i=0}^{\infty} \pi_i = 1$$

$$\pi_0 + \rho \pi_1 + \rho^2 \pi_2 + \dots = 1$$



$$\rho = 1$$

$$\sum_{i=0}^{\infty} \rho^i \pi_0 = 1$$

$$\pi_0 = \frac{1}{\sum_{i=0}^{\infty} \rho^i} = \frac{1}{1/(1-\rho)} = (1-\rho)$$

$$\pi_n = \rho^n \pi_0 = \rho^n (1-\rho)$$

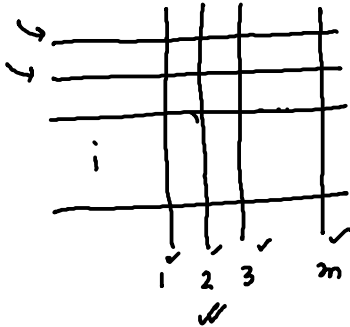
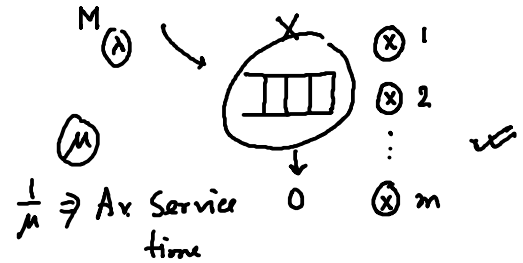
$$\bar{W} = \frac{1}{\lambda} \bar{N}$$

$$\bar{N} = \sum_{i=0}^{\infty} i \pi_i = \left(\frac{\rho}{1-\rho} \right)$$

$$\bar{W} = \frac{1}{\lambda} \bar{N} = \frac{\rho/\lambda}{1-\rho} = \frac{1/\mu}{(1-\rho)}$$

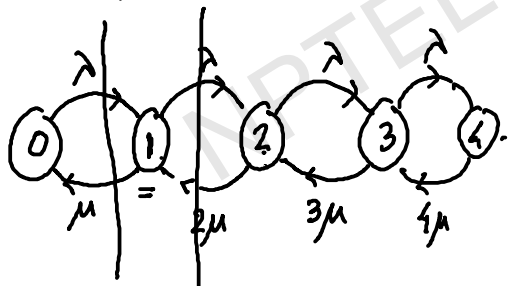
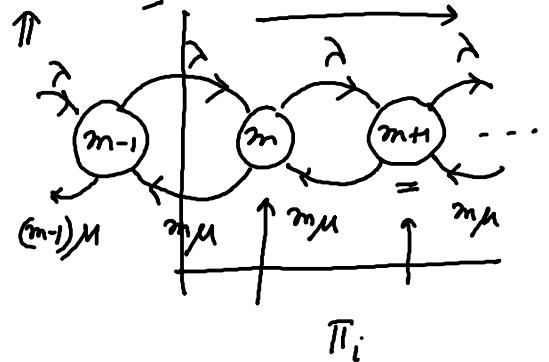
$$\rho = \frac{\lambda}{\mu}$$

$$(\mu + \mu) = 2\mu$$



⇒ Block call cleared [M/M/m/m] Erlang B model

⇒ Block call delayed [M/M/m] Erlang C model.



$$E_r[C] = P_{BCD} = \pi_m + \pi_{m+1} + \dots$$

$$\rho = \left(\frac{\lambda}{\mu} \right) \quad (m)$$

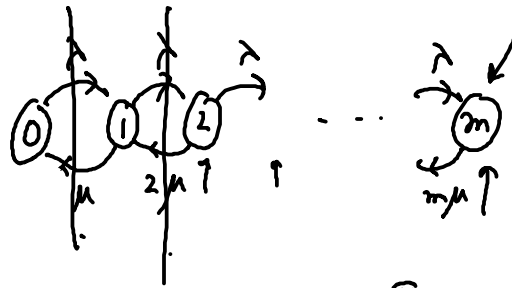
$$\lambda < \mu, \quad M/M/$$

$$\sum_{i=0}^{\infty} \pi_i = 1$$

$$\pi_0$$

$$\begin{aligned} \lambda \pi_0 &= \mu \pi_1 & \pi_1 &= \rho \pi_0 \\ \lambda \pi_1 &= 2\mu \pi_2 & \Rightarrow \pi_2 &= \frac{1}{2} \left(\frac{\lambda}{\mu} \right) \pi_1 \\ & & &= \frac{\rho^2}{1 \cdot 2} \pi_0 \\ \lambda \pi_{n-1} &= (n\mu) \pi_n & &= \frac{\rho^n}{n!} \pi_0 \\ \lambda \pi_{m-1} &= m\mu \pi_m \\ \lambda \pi_m &= m\mu \pi_{m+1} \\ & & & \vdots \\ \lambda \pi_i &= m\mu \pi_{i+1} \end{aligned}$$

M/M/m/m



$$\pi_m = \rho_B$$

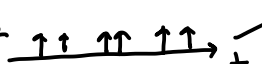
①
⋮
②

$$\sum_{i=0}^m \pi_i = 1$$

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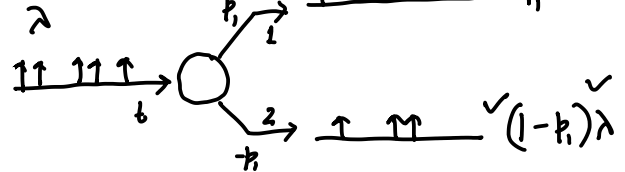
(i) PM $\rightarrow \lambda_1$  $\lambda = \lambda_1 + \lambda_2 + \dots + \lambda_n$

(ii) PS $\rightarrow$ 

(iii) MED $\rightarrow \lambda_2$ 

λ_n

n

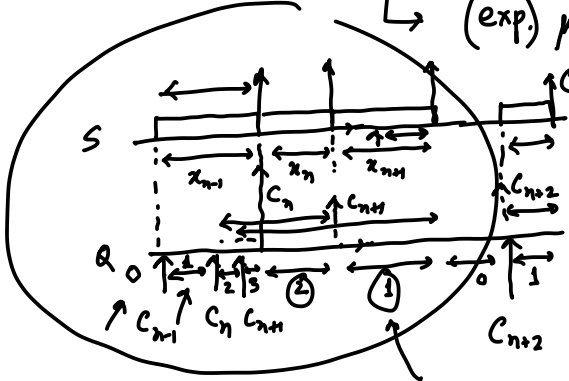
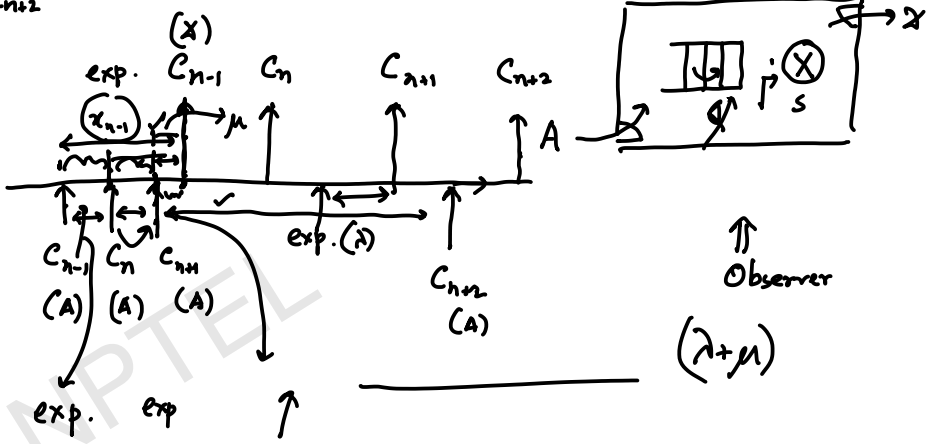


$$X = \min_i \{X_i\} \quad \forall i \in N$$

$$(\text{exp.}) \mu = \{\mu_1 + \mu_2 + \dots + \mu_n\}$$

$$N = \{1, \dots, N\}$$

$$\mu_i = \{\mu_1, \mu_2, \dots, \mu_n\}$$

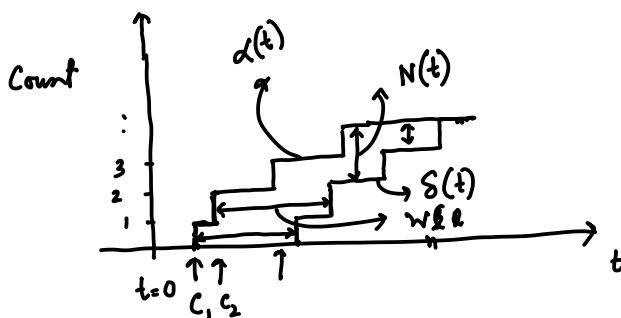


Little's Form

λ Av. arrival rate

N Av. # of customers in the system

W Av. overall time spent in the system.



$$S(t) \leq \alpha(t)$$

$$\alpha(t) - S(t)$$